

#### CGSWH

#### **General Mathematics Unit 4**

### SAC 3 2023 Matrices

Areas of study Matrices: Matrices & Transition Matrices

#### **Outcomes**

Outcome	Key knowledge dot point	Key skill dot point
1	1, 3, 4	1, 2, 3, 4
2	1, 2, 3	3, 4
3	1, 2, 7	1, 2, 3, 4

#### Materials Permitted

Students are permitted to bring into the examination room:

One bound reference, pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one approved CAS calculator and, if desired, one scientific calculator.

Students are *NOT* permitted to bring into the examination room:

Blank sheets of paper, white out liquid/tape.

Mobile phones and/or any other unauthorised electronic devices.

## An investigation into Madagascar's land use

## Introduction

Located off the east of Africa, Madagascar is the world's fifth largest island. Madagascar's animals and unique habitats are threatened by deforestation (clearing forested land). The area of Madagascar that you will study is highlighted in **red** on the map below.



The land highlighted in red above has a data set of 42 parcels. A **parcel** of land (or plot) is an area of land with a particular use, such as growing crops. Data has been collected from these 42 parcels for 22 years. Some parcels were initially forest but have since been cleared.

Each parcel can take three possible states: annual crop (A), fallow (F), perennial crop (P). These are defined as follows:

Annual crop (A) – Annual crops have a life cycle that lasts at most one year. Annual crops germinate from a seed, grow, flower and seed before dying all within a year. Annual crops need to be replanted each year.

Fallow  $(\mathbf{F})$  – A fallow is a stage of crop rotation whereby the land is deliberately not used to grow a crop, typically for one to five years. Fallow allows the soil to have a rest period to replenish nutrients and saves money on fertilisers and irrigation.

*Perennial crop* (P) – *Perennial crops do not have to be reseeded or replanted every year. After harvest, they automatically grow back.* 

**Reading Time: 5 minutes** 

Writing Time: 55 minutes Bound reference & CAS permitted



Student's Name:

**Teacher's Name:** 

### CGSWH

#### GENERAL MATHEMATICS SAC 3, 2023

#### PART 1

## An investigation into Madagascar's land use

#### Introduction

Located off the east of Africa, Madagascar is the world's fifth largest island. Madagascar's animals and unique habitats are threatened by deforestation (clearing forested land). The area of Madagascar that you will study is highlighted in **red** on the map below.



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Each parcel can take three possible states: annual crop (A), fallow (F), perennial crop (P). These are defined as follows:

Annual crop (**A**) – Annual crops have a life cycle that lasts at most one year. Annual crops germinate from a seed, grow, flower and seed before dying all within a year. Annual crops need to be replanted each year.

Fallow (F) – A fallow is a stage of crop rotation whereby the land is deliberately not used to grow a crop, typically for one to five years. Fallow allows the soil to have a rest period to replenish nutrients and saves money on fertilisers and irrigation.

Perennial crop (**P**) – Perennial crops do not have to be reseeded or replanted every year. After harvest, they automatically grow back.

This set of data collected from the 42 parcels is presented in Table 1, below.

### Raw data

	parcel number																																										
		1	2	3	4	5	6	7	8	9	10	) 11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42
	1	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f
	2	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	Α
	3	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	Α
	4	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	Α	Α	Α	А	Α	Α	А	Α	А	f	f	f	f	f	f	f	f	f	Α
	5	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	Α	Α	Α	А	Α	Α	А	Α	F	f	f	f	f	f	f	f	f	f	Ρ
	6	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	Α	A	Α	Α	Α	Α	F	F	А	f	f	f	f	f	f	f	f	f	Ρ
	7	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	Α	A	F	A	Α	F	F	Α	А	f	f	f	f	f	f	f	f	f	Ρ
	8	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	Α	A	F	Α	Α	F	А	Α	F	f	f	f	f	f	f	f	f	f	Ρ
	9	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	Α	Α	F	А	F	А	F	F	F	f	f	f	f	f	f	f	f	f	Ρ
ar	10	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	F	F	F	F	F	А	F	F	F	f	f	f	f	f	f	f	f	f	Ρ
ye	11	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	F	F	F	F	А	А	F	Α	А	f	f	f	f	f	f	f	f	f	Ρ
	12	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	Α	Α	F	F	F	F	Α	F	F	Α	Α	f	f	f	f	f	f	f	f	f	Ρ
	13	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	A	A	F	F	F	Α	Α	F	A	Α	F	f	f	f	f	f	f	f	f	f	Ρ
	14	f	f	f	f	f	f	f	A	f	f	f	f	f	f	f	f	f	f	f	f	f	A	A	F	F	F	A	Α	A	A	F	F	f	f	f	f	f	f	f	f	f	Ρ
	15	f	f	f	f	A	A	A	A	A	A	Α	f	f	f	f	f	f	f	f	f	f	A	A	F	F	F	Α	Α	F	F	Α	F	f	f	f	f	f	f	f	f	f	Ρ
	16	A	f	f	f	A	A	A	A	Α	A	Α	A	Α	Α	A	A	A	Α	A	Α	A	A	A	A	F	F	Α	F	F	F	A	A	Α	A	Α	A	A	Α	Α	Α	A	Ρ
	17	A	A	A	A	A	A	A	F	A	A	Α	A	A	A	F	F	F	A	F	A	F	F	F	Α	F	F	A	F	F	F	Α	A	Α	A	A	A	A	Α	Α	A	A	Ρ
	18	F	A	A	A	A	A	A	F	A	F	A	F	A	A	F	F	F	A	F	A	A	A	F	F	A	Α	F	A	A	F	F	F	A	A	A	A	A	F	F	A	A	Ρ
	19	F	A	A	A	A	A	Ρ	A	A	A	A	F	Α	A	F	F	F	A	F	F	A	A	A	F	A	A	F	A	Α	F	F	F	Α	A	A	A	A	F	Α	A	A	Ρ
	20	F	Α	Α	Α	Α	A	Ρ	A	A	A	Α	F	Α	Α	F	F	F	F	F	F	A	A	Α	F	F	F	Α	F	Α	F	Α	F	Α	F	Α	A	A	Α	Α	Α	A	Ρ
	21	A	P	A	A	A	F	P	A	F	F	F	F	F	F	A	A	A	F	F	F	A	F	A	A	F	A	A	F	A	A	A	A	F	A	F	A	A	F	A	A	A	P
	22	А	Ρ	A	A	A	A	Ρ	A	F	A	Α	F	F	F	A	A	F	F	F	F	F	A	F	F	A	A	A	F	A	A	F	A	A	A	A	F	A	A	F	A	Α	Ρ

Table 1. Annual states corresponding to 42 parcels and 22 years. The states are: annual crop (**A**), fallow (**F**), perennial crop (**P**) and natural forest (**f**). The f state, which will be omitted, is land still forested.

#### Using transitional diagrams and matrices to investigate the movement of crops

The case study examines the movement of different types of crops from one year to another (yearly transition). Each parcel can take three possible states: annual crop (A), fallow (F), perennial crop (P).

The transitional probabilities are estimated from observed data. The initial transition diagram detailing the movement of crops from one year to the next is shown below:



#### Questions to answer

Assume there are 42 parcels that need to be allocated to Annual (A), Fallow (F) or Perennial (P) crops.

- a) Explain why Perennial crops to perennial crops from one year to the next is 1 in the Transition diagram.
- **b)** Copy and complete the transition matrix below using the transition diagram given in Model 1 above.

This vear  

$$A$$
  $F$   $P$   
 $T = \begin{bmatrix} & & \\ & &$ 

c) Choose a year from the raw data between <u>year 17 and 22</u> when the land has been deforested (no forest land remaining). Create an initial state matrix based on the allocation of the 42 parcels between the 3 options from your chosen year. Explore how these allocations change over time. Summarise your findings. For example, for the raw data for year 15 as shown below, **ignoring forest land** (f), and

**d)** Using the transition matrix from b), explore what happens if you randomly reallocate the 42 parcels among the 3 options to create a new initial state matrix. Explore how these allocations change in the long term. Summarise your findings.

e) The researchers decide to extend the area of investigation to include a further 10 parcels of land that are added at the end of each year. Randomly allocate the 10 additional parcels to the 3 options and create a column matrix, to represent this. Use the same initial state matrix and transition matrix from previously in parts b and c. Set up a recurrence relation to model this and investigate the allocations over the first 5 years. Summarise your findings.

#### GENERAL MATHEMATICS SAC 3, 2023

#### PART 2

## An investigation into Madagascar's food chain

## Madagascar's food chain

The second part of this investigation will examine some of the animals and plants that are threatened in Madagascar's wildlife.

You will be referring to the following food chain. The arrows point to an animal/plant that is consumed (eaten) by another animal/plant.



Plant/animal name	Abbreviation
Ring-tailed lemur	Le
Tamarind	Т
Flying fox	Ff
Mango	Ma
Fossa	Fo
Nile crocodile	С
Madagascar tree boa	В
Narrow-striped mongoose	Мо
Tomato frog	Fr
Grey Mouse lemur	GLe
Lianas	Li

#### Questions to answer

**a)** Choose 8 animal and plants from the food chain above. You must include at least two plants, the Nile crocodile, and the Ring-tailed lemur in your selection. Circle your 8 selections on the diagram above. Complete the one-step dominance matrix below and clearly label the matrix with your selections <u>using the key</u> above. Provide a rank order for all listed animals and plants.



**b)** Calculate a two-step dominance matrix. Choose two non-zero elements from this matrix and list the predator-prey sequence linked to the choices made. Complete the total dominance ranking to determine if one animal/plant is dominant over your other choices. Comment on your results.

#### The ring-tailed lemur



Ring-tailed lemurs live in the forests of Madagascar. Ring-tailed lemurs are identifiable by their long, vivid black and white striped tail and golden eyes. Habitat destruction is the main threat facing ring-tailed lemurs in the wild. They are considered an endangered species (at risk of becoming extinct).

You will now investigate the population and survival of the ring-tailed lemur over time.

Table 2 contains the birthing and survival rates for age groupings of the ring-tailed lemur.

Age group ( <i>i</i> )	1	2	3	4
Age range (years)	0-3	4-7	8-11	12-15
Initial population				
Birthing rate	0	0.8	0.7	0.4
Survival rate	0.6	0.8	0.7	0

c) It is estimated that there are 2200 lemurs currently living in Madagascar. Randomly allocate the **population of 1100** female lemurs to each age group and create the initial population state matrix. Create a Leslie matrix for the birthing and survival rates of the female lemur ring tailed lemurs.

d) Create a life cycle transition diagram for the ring-tailed lemur.

e) Investigate the population of ring-tailed lemur in both the short term and long term. Summarise your findings. Select two consecutive years and compare the population growth rate for each of the 4 age groups. Comment on your findings.

# <u>Madagascar Matrices SAC – Performance Criteria Modelling/Problem Solving Task</u>

Outcome 1 Criterion (5 marks)	Marks	1	2	3	4	5
Part 1 c) Use $S_0 =$						
Part 2 a) Label rows and columns of dominance matrix						
Part 2 c) label $S_0$ and L						
Part 2 c) labels and row and column of L						
Part 1 a) interpret 1 in T,						
Part 1 b) correct elements in T						
Part 2 b) Show $T = D + D^2$						
Part 1 c) State $S_0$ correctly (42),						
Part 1 d) State $S_0$ correctly (42, reallocate),						
Part 1 e) Creating a new matrix which sums to 10,						
Part 2 a) stating dominance sum						
Part 2 b) showing numerical total dominance sum						
Part 2 c) Create $S_0$ that sums to 1100 lemurs						
Part 2 c) create correct L						
Part 2 c) correct life cycle diagram						
		Outcon	ne 1 Tot	tal		

Outcome 2 Criterion (10 marks)	Marks	1	2	3	4	5
Part 2 a) correct rank order from their one-step (D) dominance score,						
Part 2 a) Show total dominance row sum or show matrix T,						
Part 2 b) listing 2 correct 2-step dominances,						
Part 2 b) correct rank order for T.						
Part 1 c) using transitions to show $\geq$ 4 states (show rule/working),						
Part 1 e) using recurrence to show consecutive states with restocking for first 5 years (show rule/working),						
Part 2 e) using transition to show $\geq$ 4 states for short-term (show rule/working).						
Part 1 c) summarise changes over the states they have selected.						
Part 1 d) summarise changes from their SS matrices,				ļ		
Part 1 e) summarise changes to the first 5 years,						
Part 2 e) comment on short and long-term population of lemurs,						
Part 2 e) comment on growth rates.						
		Outco	me 2 To	tal		

Outcome 3 Criterion (5 marks)	Marks	1	2	3	4	5
Part 2 b) Showing $D^2$ 8 x 8 matrix						
Part 2 e) showing ratio calculation to growth rate for all 4 age groups						
Part 1 d) Showing SS with 2 consecutive same states (land),						
Part 2 e) showing SS with 2 consecutive same states (lemur)						
		Outco	me 3 To	tal		